Gibbsian representation of knowledge in infinite dimensional random neural networks(IDRNN)

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Abstract. **In studying of a class of random neural network, some of relative researchers have proposed Markov model of neural network. Wherein Markov property of the neural network is based on "assuming". To reveal mechanism of generating of Markov property in neural network, it is studied how infinite-dimensional random neural network (IDRNN) forms inner Markov representation of environment information in this paper.Because of equivalence between markov property and Gibbsian our** conclusion is that knowledge is eventually expressed by extreme Gibbs probability measure一ergodic Gibbs probability measure in IDRNN. This conclusion is also applicable to quantum mechanical level of IDRNN. Hence one can see " concept "－" consciousness" is generated at particle(ion) level in the brain and is experienced at the level of the neurons; **We have discussed also ergodicity of IDRNN with random neural potential.**

Keywords: Infinite-dimensional random neural network(IDRNN), Markov representation, Random neural potential ,Ergodic Gibbs probability measure，Extreme point,Fock space.

Ⅰ. Introduction

In studying of a class of random neural network, some of relative researchers have proposed Markov model of neural network (Emile Aarts, Jan Korst, 1989; M.M. Van Hulle and G.A. Orban, 1991; Guangcheng Xi, 1991 , and so on). It is studied how infinite-dimensional random neural network forms inner Markov representation of environmental information in this paper. Representational theory of neural network is one how environmental information is modeled in the neural network. Forming and renewing of representation of external world in neural network involve neural structure. Studying of forming mechanism for this class of representation is favourable for understanding location and mechanism of various neural structure, the class of representation is base for human brain to generate

consciousness and can constitute new algorithm or given out solution of some algorithms.

Except for brief introduction, this paper consists of three parts. In second part we suppose S is countable infinite set of neurons coupled by synapses to one another, ($E, B(E)$) and ($X = E^S, B(X)$) two measurable spaces, where E is any measurable set of all possible values taking by single neuron. By means of analyzing generating process of so-called limit Markov process in IDRNN, we have shown in limit state of process for IDRNN to learn of environment (i.e. neuron in group of neurons $C \subset S$ takes a given value $y_b = \bar{y}_b \in E$) the limit Markov process is generated in IDRNN, this is to say, environment-knowledge is expressed by means of the limit Markov process in IDRNN, have pointed out set of all stationary distribution of the limit Markov process on IDRNN—set of limit Gibbs probability measure is a nonempty closed convex set of which ergodic Gibbs probability measure is extreme. Knowledge is eventually expressed by extreme Gibbs probability measures in IDRNN. In third part of this paper, we have discussed ergodicity of IDRNN with random neural potential. Fourth part is a remark.

II. Markov representation of knowledge in IDRNN

Suppose S is a countable infinite set of neurons coupled by synapses to one another, ($E, B(E)$) and ($X = E^S, B(X)$) two measurable spaces. Where E is any measurable set of all possible values taken by single neuron. Hence, we have infinite dimensional random neural network (IDRNN).

In studying of human train's high function—consciousness, one suggested that a key neural mechanism underlying conscious experience are the recentrant interactions between posterior thalamocortical areas involved in perceptual categorization and anterior areas related to memory, value and planning for action. Such interactions among neural groups in distributed brain areas may be necessary in order to generate a unified neural process corresponding to a multimodal conscious scene (G. Tononi and Edelman. G. M., 1998).In neurophysiological research of information representation in the cortex, a possible neural coding mechanism of spatiotemporal firing pattern of neurons is based on the temporal cell subassemblies which spontaneously dynamically organize into a dynamical cell assembly by synchronized activity of these subassemblies.Hebb's viewpoint has considered the assembly of neural cell has played key role in neural coding and neural information representatioin. Therefore, in the IDRNN

by contrast to probability $p(x,t)$ that IDRNN is in state $x \in X$ at the moment t, we have more interested in total probability (or its limit) that neurons in neural group $C \subset S$ take given value $\bar{y}_b \in E, b \in C$ at the moment t. Such total probability and its limit are expressed, respectively, as

$$p_C(\bar{y}_b, b \in C, t) = \sum_{x:y_b = y_b, b \in C} p(x,t), \tag{1}$$

$$\hat{p}_C(\bar{y}_b, b \in C) = \lim_{t \to \infty} p_C(\bar{y}_b, b \in C, t) = \sum_{x:y_b = y_b, b \in C} \hat{p}(x). \tag{2}$$

Suppose for some $\bar{x} \in X$ and any $x \in X$ and $l < \infty$, can find finite sequence $x = x_1, x_2, \cdots, \bar{x} = x_l$ such that

$$p(x_1, x_2) p(x_2, x_3) \cdots p(x_{l-1}, x_l) > 0 \ , \tag{3}$$

where $p(x_i, x_j), 1 \le i, j \le l$ is transition probability from x_i to x_j, then there exists limit

$$\hat{p} = \lim p(x,t), \qquad x \in X \tag{5}$$

independent of initial distribution $p(0)$. Probability distribution $\hat{p} = \{\hat{p}(x), x \in X\}$ is called stationary, if $p(0) = \hat{p}$, then $p(t) = \hat{p}, 0 \le t < \infty$.

Let us suppose that

(i)   The state change of the IDRNN only permits one neuron to change own state at a time.

(ii)   The set S of neuron lies in d —dimensional integer lattice R^d. $A \subset R^d$ is a cube with side $2L$ in R^d, i.e. $a \in A$ such that $a = (a_1, a_2, \cdots, a_d)$, where a_i is integer, $|a_i| \le L, i = 1, 2, \cdots, d$. Radius of interaction between neurons is r, i.e., if $B_b = \{c : c \in A, |b - c| \le r\}$ is a sphere with center b, then when $x = (y_a, a \in A) \in E^A$, we have

$$p_b(x, \bar{x}) = p_b(x, \bar{y}_b) = p_b(\bar{y}_b \mid y_a, a \in B_b) \ , \tag{5}$$

if $B_b \subset A$. Where $p_b(x, \bar{y}_b)$ is probability that over time dt IDRNN lying in state x changes state of own neuron b from y_a to $\bar{y}_b$. Condition (5) means that probability density giving change of state of neuron b is only dependent on states of neurons in IDRNN which are not more than r distant from b.

In later description, the IDRNN will satisfy above-mentioned two conditions.

For any neuron $b \in R^d$, density of transition probability $p_b(\bar{y}_b \mid y_a, a \in B_b)$ is given and satisfies inequality

$$0 \le p_b(\bar{y}_b \mid y_a, a \in B_b) \le K \ , \tag{6}$$

for all $b \in R^d$, $\bar{y}_b \in E, y_a \in E, a \in B_b$ and $K < \infty$. Set of all density of transition probability $\{p_b(. \mid .), b \in R^d\}$ is called a family of density of transition probability and denoted by Λ. Restriction on cube $A \subset R^d$ of the process $\{x_b(t), t \in T, b \in R^d\}$ whose family of

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transition probability is Λ is a process $\{x_b(t), t \in T, b \in A\}$ having finite number of state, if $B_b \subset A$, then when $x = (y_a, a \in A)$, following expression

$$p_b(x, \bar{y}_b) = p_b(\bar{y}_b \mid y_a, a \in B_b) \qquad x \in E^A, \quad \bar{y}_b \in E \quad , \tag{7}$$

holds. When cube A is given, the restriction remains not uniquely be determined. Since for those $b \in A$, for which distance from b to boundary of cube A is smaller than r, possibilities of choosing the density by different manner still will exist. Such density is called boundary density, also satisfies following inequality

$$0 \leq p_b(x, \bar{y}_b) \leq K, x \in E^A, \bar{y}_b \in E, b \in A . \tag{8}$$

If $p_b(x, \bar{y}_b)$ only dependent on $y_a \in B_b \cap A (b \in A)$, then we will call such boundary density local. Most natural method to give local boundary density is to set

$$p_b(x, \bar{y}_b) = p_b(\bar{y}_b \mid y_a, a \in B_b \cap A, y_a^0 \in E, a \in R^d \setminus A) \quad , \tag{9}$$

having fixed values $y_a^0 \in E, a \in R^d \setminus A$ by any methods. When expression (9) is carried out, $(y_a^0 \in E, a \in R^d \setminus A)$ is called constant boundary condition.

Proposition 1（Dobrushin R.L.,1971）.

Suppose for any $L = 1,2,\cdots\cdots$, the restriction of neural process $\{x(a,t), a \in R^d, t \in \tau(\text{set of time})\}$ with family of density Λ on cube A_L with side *2L* is given. Let $\hat{x}_0 = (y_a^0, a \in R^d)$ ——arbitrary collection of $y_a^0 \in E$, and $p_x^L(t \mid x_L^0), x \in X_L, 0 \leq t \leq \infty$, where $X_L = E^{A_L}$ ——space of state of the restriction on A_L, be probability of state x on the restriction at moment *t*. If initial distribution is chosen so that

$$p_{x_L^0}^L(0 \mid x_L^0) = 1, \quad x_L^0 = (y_a^0, a \in A_L) \in X_L \quad , \tag{10}$$

thus, if $C \subset A_L$, $\bar{y}_b \in E, b \in C$ and under inspiration of expression (1) and (2), we have

$$\mu_C^L(\bar{y}_b, b \in C, t \mid x_L^0) = \sum_{x: y_b = \bar{y}_b, b \in C} p_x^L(t \mid x_L^0) \quad . \tag{11}$$

Then, for any *t*, $0 \leq t < \infty$, any finite $C \in R^d$ and any $\bar{y}_b \in E, b \in C$, limit

$$\mu_C^\infty(\bar{y}_b, b \in C, t \mid x_L^0) = \lim_{L \to \infty} \mu_C^L(\bar{y}_b, b \in C, t \mid x_L^0) \quad , \tag{12}$$

there exists.

In IDRNN, set of probability $\{\mu_C^\infty(\bar{y}_b, b \in C, t \mid x_L^0), C \subset R^d$, $\bar{y}_b \in E, b \in C\}$ gives consistent family of finite distribution for random field $\{x(b,t), b \in R^d, t \in \tau\}$, and so, in accordance with well known Kolmogorov's theorem, gives probability measure $\mu(\cdot, t \mid \hat{x}_0)$ which is measurable with respect to $\hat{x}_0$ on measurable space $(X = E^S, B(X))$. Also $\mu(\cdot, t \mid \hat{x}_0)$ satisfies following Komogorov-Chapman equation

$$\mu(., t+s \mid \hat{x}_0) = \int_X \mu(\cdot, s \mid \hat{x}) \mu(d\hat{x}, t \mid \hat{x}_0) , \quad \hat{x}_0 \in X, 0 \leq s < \infty, 0 \leq t < \infty. \tag{13}$$

Hence probability measure $\mu(\cdot, t \mid \hat{x}_0)$ is transition probability. In accordance with Regular Markov existence theorem(M. Loève, 1978, II)，the transition function $\mu(\cdot, t \mid \hat{x}_0)$ gives time-homogeneous Markov process in the IDRNN. It is limit Markov process.

Now we discuss correctness of expression (12).

Definition 1. (K.R.Parthasarathy, 1977).A measure μ on a metric space X is said to be tight if for each $\varepsilon > 0$ there exist a compact set $K_\varepsilon \subseteq X$ such that $\mu(X - K_\varepsilon) < \varepsilon$.

Definition 2. A measurable space $(E, B(E))$ is called a standard Borel space if there exists a metric d on E which turns E into a complete separable metric space and is such that $B(E)$ is the Borel σ-algebra with respect to d.

Lemma 1 (K.R.Parthasarathy, 1967). Supposed metric space X is a complete separable, then every measure on X is tight.

In IDRNN, suppose E is standard Borel space. Thus measurable space $(X = E^{R^d}, B(X))$ is also standard Borel. From the two definitions 1,2 and Lemma 1, having considered subtractness of probability measure (i.e. $\mu(X - K_\varepsilon) < \varepsilon$ can be expressed as $[\mu(X) - \mu(K_\varepsilon)] < \varepsilon$), we known that there exists a number N such that

$$\left| \mu_C^L(\cdot \mid \cdot) - \mu_C^\infty(\cdot \mid \cdot) \right| < \varepsilon, \tag{14}$$

with $L>N$, i.e., sequence $\{\mu_C^L\}(L=1,2,\cdots)$ converges to $\mu_C^\infty(\cdot \mid \cdot)$, $\lim_{L \to \infty} \mu_C^L(\cdot \mid \cdot) = \mu_C^\infty(\cdot \mid \cdot)$.

From preceding description, we showed in limit state of process for IDRNN to learn of environment (i.e. neuron in group of neuron $C \subset S$ takes a given value $y_b = \bar{y}_b \in E$), the limit Markov process is generated in IDRNN. That is to say, environment-knowledge is expressed by means of limit Markov process in IDRNN. The limit Markov process is a limit behavior of learning process for IDRNN .

Definition 3. For any $A_L = (a_L^1, \cdots, a_L^m) \subset R^d$, any $x_i \in E$, $i = 1,2,\cdots,m$ and any real function $x(a), a \in R^d \setminus A_L$, probability distribution

$$\lambda_{A_L}(x_i, i = 1,2,\cdots,m \mid x(a)) = \exp\{-\beta U_{A_L}(x_i, i = 1,2,\cdots,m) \mid x(a)\}[\Phi_{A_L}(x(a))]^{-1} \tag{15}$$

is known as Gibbs distribution with boundary condition $x(a)$ in A_L, where

$$\Phi_{A_L}(x(a) = \sum_{x_i \in E, i=1,\cdots,m} \exp\{-\beta U_{A_L}(x_i, i = 1,2,\cdots,m) \mid x(a)\} \tag{16}$$

is called partition function; $0 < \beta = 1/T < \infty$; U_{A_L} is called neural potential, it is defined as

$$U_{A_L}((x_i, i=1,2,\cdots,m) \mid x(a)) = -c\sum_{i=1}^m x_i + \frac{1}{2}\sum_{i=1}^m \sum_{j=1, j \neq i}^m x_i x_j U_{A_L}(a_i - a_j) + \sum_{i=1}^m \sum_{a \in R^d \setminus A_L} x_i x(a) U_{A_L}(a_i - a), \tag{17}$$

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where c is a constant, the chemical potential.When $L \rightarrow \infty$,we have

$$\lim_{L \rightarrow \infty} \lambda_{A_L}(\bullet|\bullet) = \lambda(\bullet|\bullet) = \exp\{-\beta U(x_i, i = 1, 2, \cdots\cdots, m \mid x_a)[\Phi(x_a)]^{-1}. \qquad (18)$$

Under neighboring situation, IDRNN is Markovian if and only if its limit probability measure is Gibbsian(John G. Kemeny et. al, 1992). By means of the Markov process, also i.e. by means of Gibbs probability measure, IDRNN expresses environment-knowledge.

Set of all stationary distribution of the Markov process on IDRNN—set of limit Gibbs probability measure is a nonempty compact convex set which ergodic Gibbs probability measure is extreme of (H.O. Georgii, 1988). Knowledge is eventually expressed by extreme Gibbs probability measure—ergodic Gibbs probability measure in IDRNN (Xi Guangcheng, 2003). Assume set of all stationary distribution in IDRNN is denoted by $M(\mu)$,then we have: *Suppose $\mu_i, i = 1, \cdots, n$, are n extreme points (n—dimesional vector), denote n concepts memorized by IDRNN. Then any concept v having n characteristic can be uniquelly expressed as convex combination of $\mu_i, i = 1, \cdots, n$,*

$$v = \sum_{i=1}^{n} \alpha_i \mu_i, \ \sum_{i=1}^{n} \alpha_i = 1. \qquad (19)$$

Under continuous case, we have

$$v = \int_{exM(\mu)} \mu\omega(d\mu) \qquad (20)$$

where $\omega \in \mathcal{P}(exM(\mu), B(exM(\mu)), \mathcal{P}(.,.)$ denotes set of all probability measure on measurable space $(.,.)$ (Xi Guangcheng, 2003).

- The above conclusion is also applicable to quantum mechanical level of IDRNN.Suppose that a neuron is excited whenever a quantum particle is located inside the neuron.Having given a single particle Hilbert space H ,let $\mathcal{F}$ be the boson Fock space corresponding to IDRNN.For each $\psi \in H$ define a unit vector $\tilde{\psi} \in \mathcal{F}$, called a coherent state.For space of measure $(H, B(H), \sigma)$ there exists the states ρ_σ on the Fock space

$$\rho_\sigma = \int_H |\tilde{\psi}><\tilde{\psi}| \sigma(d\psi). \qquad (21)$$

The states ρ_σ are called classical states and form a convex subset of the state space V of $\mathcal{F}$. In certain conditions ρ_σ is Gibbsian. In addition, the set L of

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linear combination

$$\sum_{l=1}^{n}\lambda_{l}\,|\,\tilde{\psi_{l}}><\tilde{\psi_{l}}\,|\,, \tag{22}$$

where $\lambda_{l} \in \mathrm{R}$ (real number) and $\psi_{l} \in H$,is dense in V for the trace norm

topology.(E.B.DAVIES,1976)

Hence one can see " concept "– " consciousness" (Xi Guangcheng, 2003; Xi Guangcheng, **2007**) is generated at particle(ion) level in the brain and is experienced at the level of the neurons from the above discussion.

Ⅲ. **Ergodicity of IDRNN with random neural potential**

Definition 4. Sequence μ_n of probability distribution on *X* is called converging to distribution $\tilde{\mu}$, if for any finite $C \subset R^d$ and any $\overline{y}_b \in E, b \in C$,

$$\lim_{n\to\infty}\mu_{n}(\{\hat{x}: y_{b} = \overline{y}_{b}, b \in C\}) = \tilde{\mu}(\{\hat{x}: y_{b} = \overline{y}_{b}, b \in C\}) \ . \tag{18}$$

Definition 5. The limit Markov process is called ergodic, if there exists probability distribution $\tilde{\mu}(\bullet)$ on *X* such that for any $\hat{x} \in X$,

$$\lim_{t\to\infty}\mu(\bullet,t\,|\,\hat{x}) = \tilde{\mu}(\bullet)\,, \tag{19}$$

in sense of definition of converging of expression (18).

Proposition 2.

Suppose $U: X \to R$ is a random transformation, R set of all real number; $U(x)$ is neural potential function corresponding to $x \in X$. Thus we have probability space $(U, B(U), Q)$. Let $\mu(\bullet, t \,|\, \hat{x})$, $\hat{x} \in X$ be transition function of the limit Markov process on IDRNN. Then there exists transition function $q(\bullet, t \,|\, \hat{u})$ induced by Q such that $q(\bullet, t \,|\, \hat{u}) = \mu(\bullet, t \,|\, \hat{x})$.

Proof of Proposition 2 relies on following two lemmas.

Lemma 1(K. R. Parthasurathy, 1977). Let *X* be complete and separable metric space $\mu(\{x\}) = 0, x \in X; ([0,1], B[0,1], l)$ probability space. Thus probability space $(X, B(X), \mu)$ is isomorphic to $([0,1], B[0,1], l$).

Lemma 2(K. R. Parthasurathy, 1977). Let $Q(\{y\}) = 0, y \in R$. Then probability space $(U = R, B(U), Q)$ is isomorphic to probability space $([0,1], B[0,1], l$).

From lemma 1 and lemma 2 and having considered transitivity of the isomorphism, we find probability space $(X, B(X), \mu)$ is isomorphic to the probability $(U = R, B(U), Q)$ and further find proof of proposition 2. From proposition 2, we have following

Corollary 1. Suppose $U = \{U_{L(t)}, L(t) \subset R^d, \lim_{t \to \infty} L(t) = R^d\}$ is Markov process on probability space $(U = R, B(U), Q)$ whose transition function is $q(\bullet, t \mid \hat{u})$. Then the limit Markov process on IDRNN is ergodic if and only if Markov process $U = \{U_{L(t)}, L(t) \subset R^d, \lim_{t \to \infty} L(t) = R^d\}$ is ergodic.

Corollary 2. （**Bassalygo,L.A. and Dobrushin,R.L.**）**.** For IDRNN with random neural potential there exist such constants *M*>0 and *m*>0, such that if

$$Q(|U_{L(t)}| > M) < m \tag{20}$$

then for Q—a.e. realization of U, IDRNN with random neural potential U is ergodic.

Proof. From definition of probability distribution Q on probability space $(U = R, B(U), Q)$ and formula (15), the proof is completed.

Corollary 3.（**Bassalygo,L.A. and Dobrushin,R.L.**）**.** For IDRNN with random neural potential, if $U = \{U_{L(t)}(x_{L(t)})\}$ satisfies uniformly following formula

$$\lim_{d \to \infty} Q(|U_{L(t)}| > d) = 0 \tag{21}$$

for $L(t)$ and $x_{L(t)}$, then there exists such β_0 that for almost all $\beta \geq \beta_0$ and almost realization of all neural potential $U_{L(t)}$, IDRNN with random neural potential $U_{L(t)}$ is ergodic.

Proof. From corollary 2 and formula (15), the proof is completed.

By the isomorphism between the state space of IDRNN with random neural potential and the space of its potential function, the studying of ergodicity of IDRNN with random neural potential is turned into the studying of ergodicity of the potential function, the resulting propositions shall be applied to the theory of ergodicity of IDRNN with random neural potential.

Ⅳ. Remark

We note that:

(1) Probability space $(X, B(X), \mu)$ is isomorphic to probability space $(V, B(V), \nu)$ on the Fock space $\mathscr{F}$, where V is state space of $\mathscr{F}$. Perhaps that is mathematical foundations of Gibbs sameness of representation of knowledge at quantum mechanical level and level of neurons in IDRNN.

(2) Let set of linear bounded operator(signal) on the Fock space $\mathcal{F}$ be von Neumann algebra ,consider quantum mechanical dynamical system C^*-dynamic system$(A, S(A), \alpha(R))$,where A is von Neumann algebra, $S(A)$ is the set of all normal states on A , $\alpha(R)$ is a strongly continuous one─parameter group of automorphism of A. Let $(X, B(X), \mu)$ be probability space on compact Hausdorff space, $x \in X \to \omega_x \in S(A)$ $B(X)$─measurable, then Λ^* defined by

$$\Lambda^* \mu = \int_X \omega_x \mu d(x), \mu \in M(\mu)$$

said to be a channel from $M(\mu)$ to $S(A)$, and channel Λ^* from $S(A)$ to $M(\mu)$ is defined by

$$\Lambda^* \phi(\,.\,) = \phi(m(\,.\,)) \qquad \phi \in S(A),$$

where m is positive $A-valued$ measure (POVM) from $B(X)$ to A with $m(Q) = \Lambda(1_Q)$,$Q \in B(X)$,1_Q is indicator function(A.S. HOLEVO,1977).

The theory provided in present article has been used already innovatively (Guangcheng Xi and Jianxin Chen,2007).

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